

# Closed incompressible surfaces of genus two in 3-bridge knot complements

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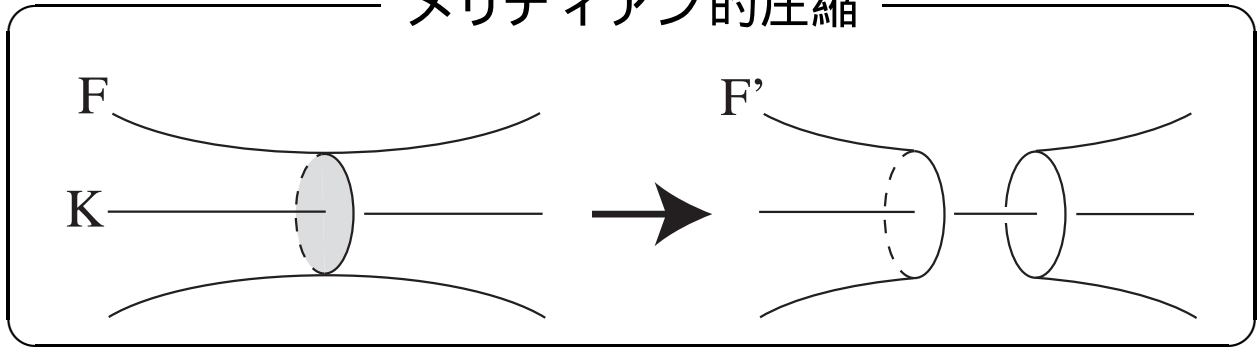
$K \subset S^3$  : 結び目

$F \subset S^3 - K$  : 圧縮不可能閉曲面 ( $\neq \partial N(K)$ )

$g(F) \backslash b(K)$	1	2	3	4	5	
1	$\phi$	$\phi$	$\phi$	$\phi$	$\phi$	
2	$\phi$	$\phi$	$\phi$	$\phi$	$\phi$	← Hatcher -Thurston
3						← This work
4						
5						

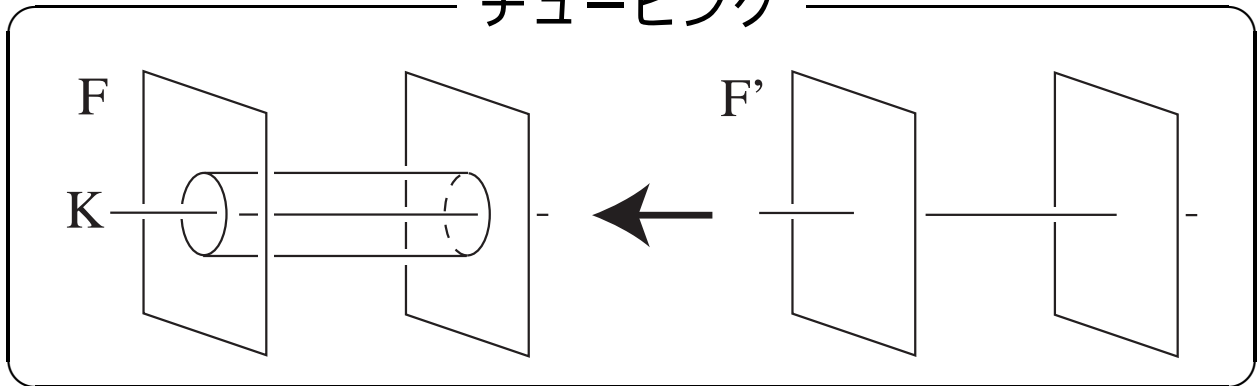
↑  
Schubert

メリディアン的圧縮



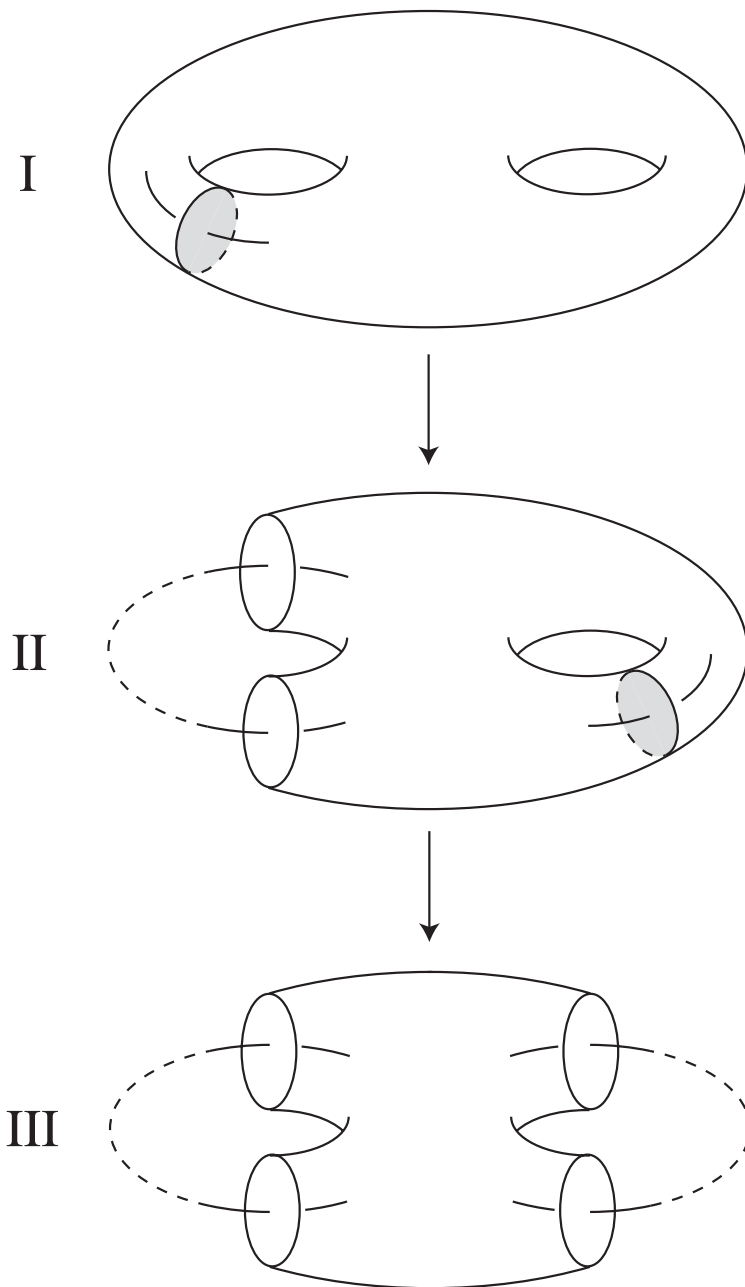
$F$  が圧縮不可能  $\Rightarrow F'$  も圧縮不可能

チュービング



$F'$  が圧縮不可能  $\Rightarrow F$  も圧縮不可能とは限らない

$F$ が種数2の場合

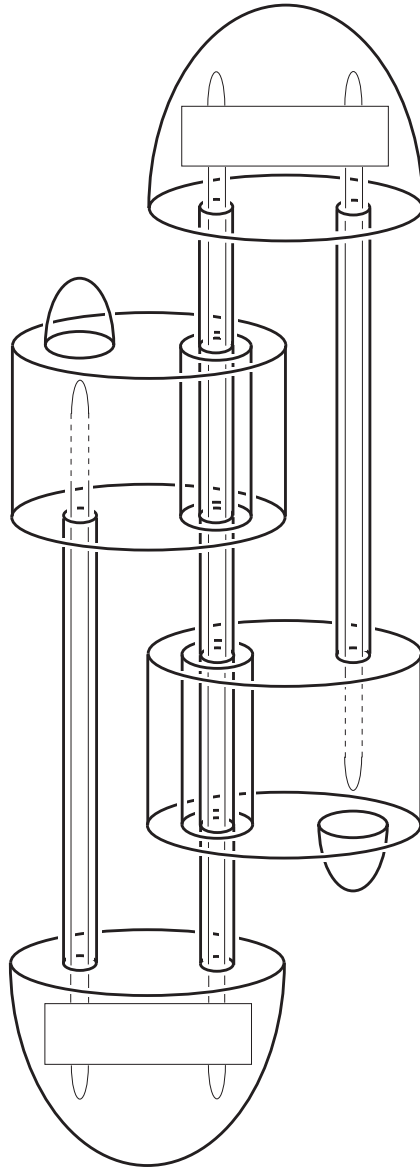


定理 I

$K$  : 3-bridge knot

$F \subset S^3 - K$  : closed incompressible and meridionally incompressible surface of genus two

$\Rightarrow$



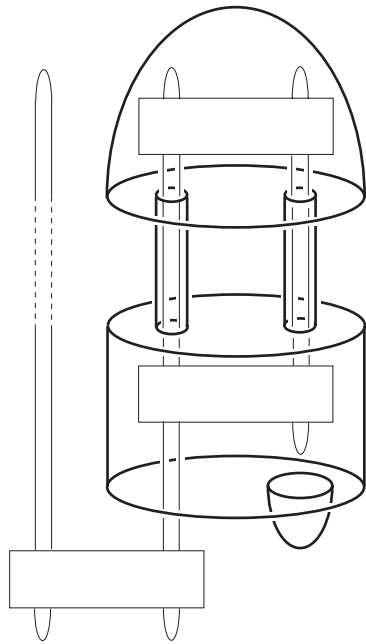
I-a

定理 II

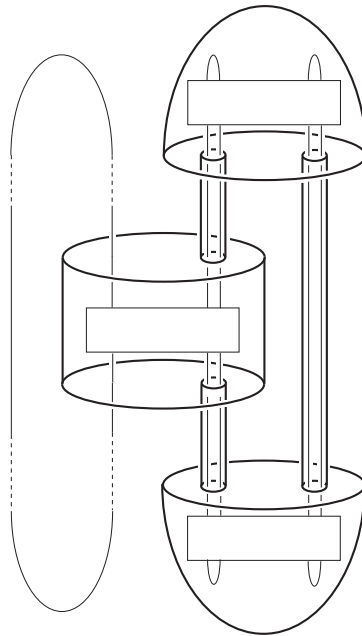
$K$  : 3-bridge knot

$F \subset S^3 - K$  : closed incompressible and meridionally incompressible surface of genus one

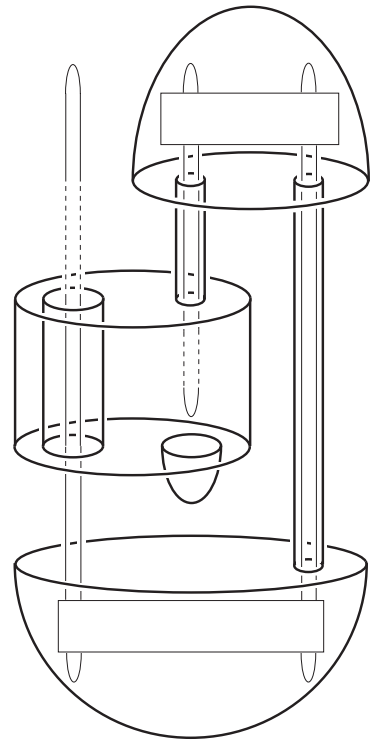
$\Rightarrow$



II-a



II-b



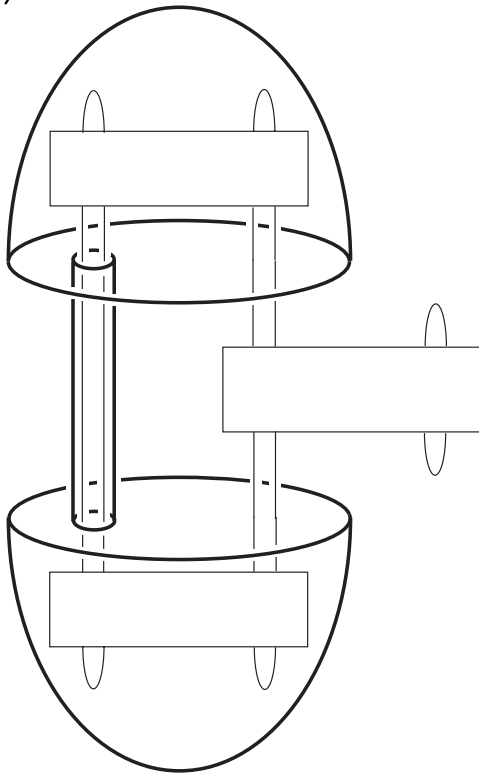
II-c

定理 III

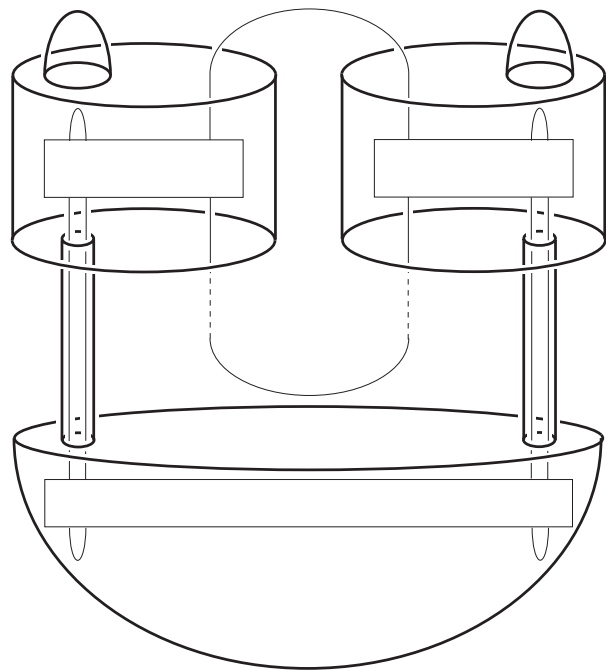
$K$  : 3-bridge knot

$F \subset S^3 - K$  : closed incompressible and meridionally incompressible surface of genus zero

$\Rightarrow$



III-a

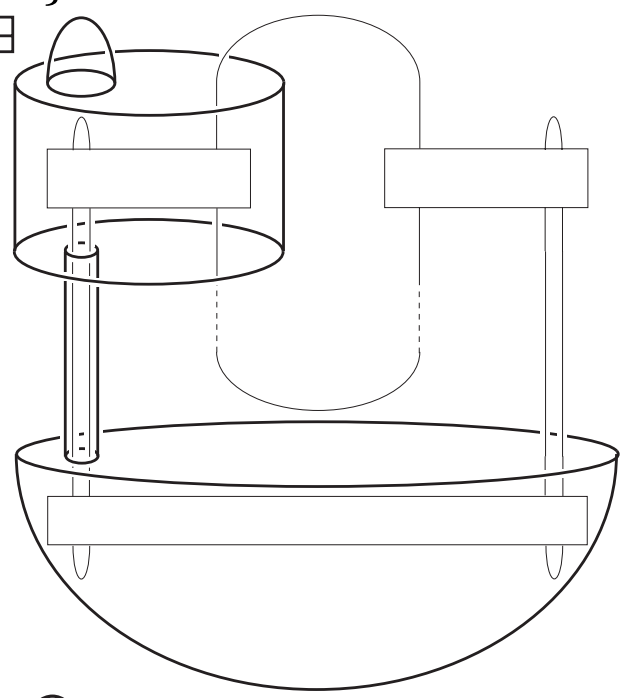


III-b

$\{\text{III-b}\} \subset \{\text{III-a}\}$

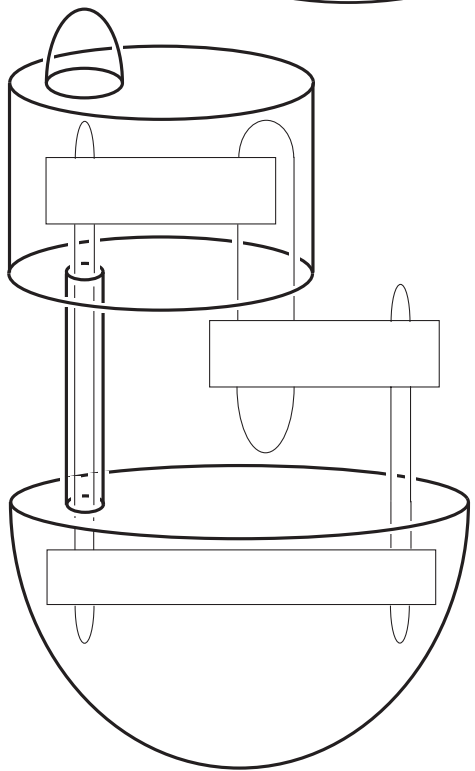
$\exists$

III-b  $\Rightarrow$



~

$\subset$  III-a





## 種数が 3 以上の場合

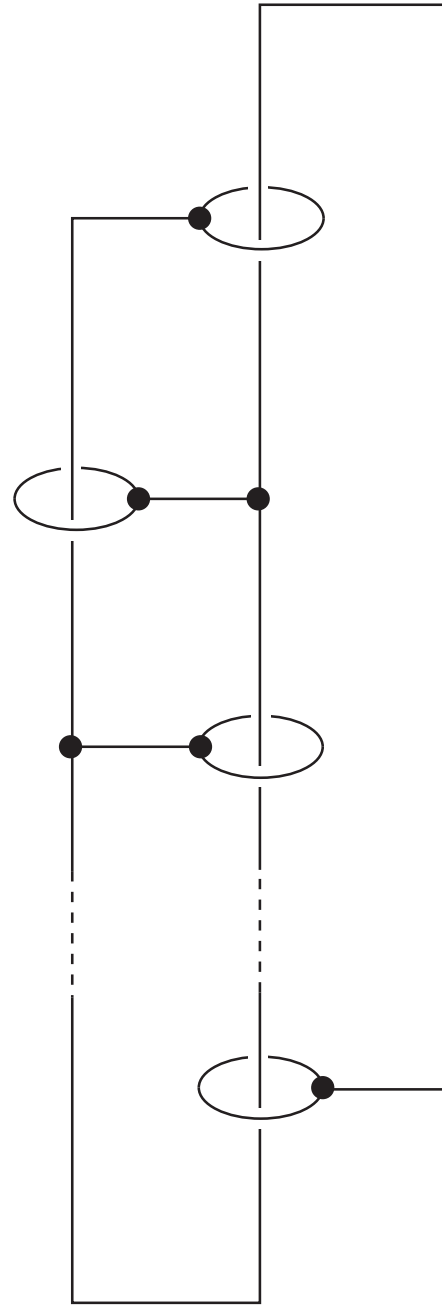
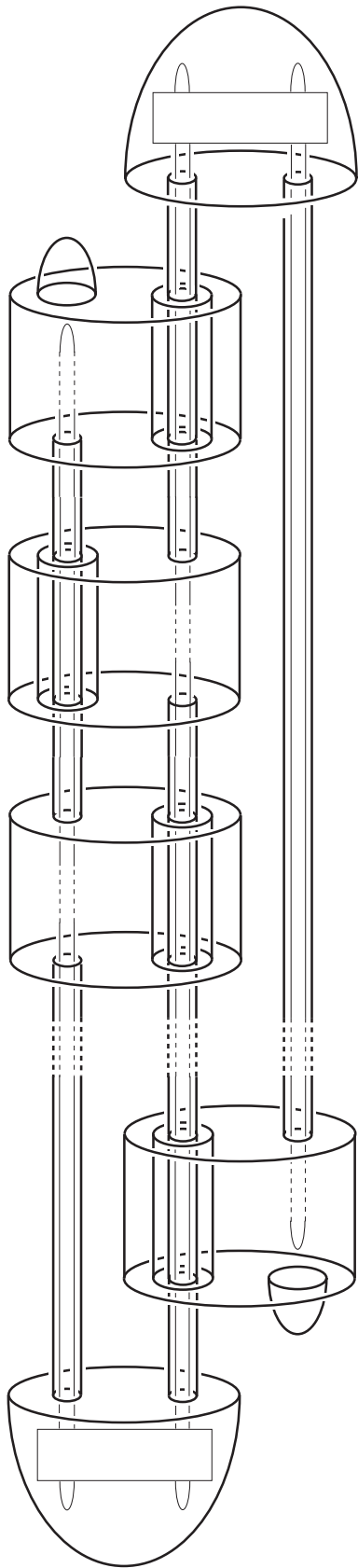
問題 1.27 ( 林 ) [低次元トポロジー問題集 1996 年]  
3-bridge knot complement 内に種数 3 以上の圧縮不可能かつメリディアン的圧縮不可能な閉曲面は存在するか？

### 定理 IV

$\forall g \geq 2$

$\exists K$  : 3-bridge knot

$\exists F \subset S^3 - K$  : closed incompressible and meridionally incompressible surface of genus  $g$



totally knotted  
spatial graph